

Taylorův a Maclaurinův polynom

Maclaurinův polynom

Maclaurinův polynom funkce $f(x)$ (v bodě 0) je:

$$M_n(x) = f(x_0) + \frac{f'(x_0)}{1!}x + \frac{f''(x_0)}{2!}x^2 + \dots + \frac{f^{(n)}(x_0)}{n!}x^n = \sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!}x^k$$

Maclaurinovy rozvoje známých funkcí jsou:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} = \sum_{k=0}^n \frac{x^k}{k!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^{n+1} \frac{x^{2n}}{(2n)!} = \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \frac{x^n}{n} = \sum_{k=0}^n (-1)^k \frac{x^{k+1}}{(k+1)}$$

$$(1+x)^a = 1 + \binom{a}{1}x + \binom{a}{2}x^2 + \dots + \binom{a}{n}x^n = \sum_{k=0}^n \binom{a}{k}x^k$$